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論　　文　　名　　Stochastic differentiability, Ogawa integrability and identification from SFCs of random functions
乱関数の確率微分可能性, Ogawa 積分可能性, 及び SFC による同定

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Stochastic differentiability, Ogawa integrability and identification from SFCs of random functions

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Abstract

We take up three topics in noncausal calculus, which is stochastic analysis for non-causal functions, and give results concerning each topic as follows:

1. Stochastic differentiability: we introduce the notion of a stochastic differential of a noncausal function and give a result on the stochastic differentiability. We briefly summarize the result. Let $V : [0, \infty) \times \Omega \to \mathbb{C}$ be a right-continuous with left-limits process with right-continuous quadratic variation process $[V]$ and with the the following property: for any interval $[s, t] \subset [0, L]$ we have $[V]_s = [V]_t$ a.s. implies ($V_s = V_u$ a.s.) for any $u \in (s, t]$. We introduce the $L^0(\Omega)$-module $\mathscr{D}(V)$ of stochastic processes X with differential $\mathcal{D}_V X$ with respect to V. Roughly speaking, the stochastic differentiation $\mathcal{D}_V$, defined as a $L^0(\Omega)$-module homomorphism, becomes the inverse operation of the stochastic integral and the space of integrands in $\mathscr{D}(V)$ of a stochastic integral forms the class of integrands of a version of the stochastic integral. We define the subset Q_V of $\mathscr{D}(V)$ and prove Q_V becomes a sub $L^0(\Omega)$-module of $\mathscr{D}(V)$. As a consequence we have the following (1) and (2) for any $X, Y \in Q_V$ and $\alpha, \beta \in L^0(\Omega)$: (1) $[\alpha X + \beta Y]_\cdot = \int_0^\cdot |\alpha\, \mathcal{D}_V X + \beta\, \mathcal{D}_V Y|^2\, d[V]$. (2) $\langle X, Y\rangle_\cdot = \int_0^\cdot \mathcal{D}_V X\, \mathcal{D}_V Y\, d[V]$. Here $\langle , \rangle$ and $[\ ]$ stand for the cross variation and quadratic variation, respectively.

2. Ogawa integrability: we give results on the Ogawa integrability, which has been studied by S. Ogawa and J. Rosinski. The Ogawa integral is one of the integrals defined for noncausal functions. Here, studying integrability means examining (necessary and) sufficient conditions for a function to be integrable and investigating how representation the integral has. We compare our results on the Ogawa integrability with previous results. Ogawa obtained theorems on the Ogawa integrability of quasi-martingales. As extensions of those theorems, we obtained teorems on the integrability of Skorokhod integral processes. Moreover we got theorem on the integrability of S-type Itô processes or more general Wiener functionals, as an extension of those theorems we obtained. On the other, Ogawa showed an integration by parts formula for the Ogawa integral. We proved this type of formula as its extension.

3. Identification problem from stochastic Fourier coefficients: we give results on the identification problem from stochastic Fourier coefficients (SFCs in abbr.),

which is another topic posed and studied by Ogawa in noncausal calculus. We explain the problem of identification. Let $(e_n)_{n\in\mathbb{N}}$ be a complete orthonormal system (CONS in abbr.) of $L^2([0,L]\,;\mathbb{C})$, $(B_t)_{t\in[0,\infty)}$ a real Brownian motion on a probability space $(\Omega,\mathcal{F},P)$ and a,b measurable functions on the product measure space $[0,L]\times\Omega = ([0,L]\times\Omega,\mathcal{B}([0,L])\otimes\mathcal{F},\lambda|_{\mathcal{B}([0,L])}\otimes P)$ taking values in $\mathbb{C}$, which we call random functions. Here, the symbol $[0,L]$ is regarded as the interval $[0,\infty)$ if $L=\infty$. We consider the SFC $(e_n,dY) := \int_0^L \overline{e_n(t)}a(t)\,dB_t + \int_0^L \overline{e_n(t)}b(t)\,dt$, where $\overline{e_n(t)}$ denotes the complex conjugate of $e_n(t)$, of the stochastic differential $dY_t = a(t)\,dB_t + b(t)\,dt$ with respect to $(e_n)_{n\in\mathbb{N}}$, which is introduced by Ogawa. We note that the SFC (e_n,dY) does not make sense unless the stochastic integral $\int_0^L dB$ is specified, $\overline{e_n}a$ is stochastic integrable and $\overline{e_n}b$ is Lebesgue integrable on $[0,L]$. Specifically, the SFC is called of Skorokhod type (SFC-S) if the stochastic integral $\int_0^L dB$ is the Skorokhod integral and of Ogawa type (SFC-O) if the stochastic integral $\int_0^L dB$ is the Ogawa integral. The question is as follows: Letting $\Lambda=\mathbb{N}$ or $\Lambda\subset\mathbb{N}$, is the map which associates a pair (a,b) of random functions a and b with the sequence of SFCs $((e_n,dY))_{n\in\Lambda}$ injective? If yes, how is the inverse of the map ?

The following are the results we obtained: to begin with we introduce the notion of constructive identification in an assigned first-order language and introduce B-dependent (resp. B-independent) identification, which can be called identification "in need of" (resp. "in no need of") the condition that the underlying Brownian motion is $(B_t)_{t\in[0,\infty)}$. We obtained affirmative answers to the question about the determinability of random functions from SFCs. The results are given in the following way. Here, the results starting with 'Derivation' give concrete derivation formulas of random functions.

- (S) Identification from SFC-Ss
  - (S-I) B-independent identification
    - (S-I-1) Theorem: Derivation of $|a|$ from SFC-Ss of a stochastic differential whose diffusion coefficient $a(t)$ is a locally absolutely continuous Wiener functional and drift term $b(t)$ is square integrable almost surely
  - (S-D) B-dependent identification
    - (S-D-1) Theorem (extension of a theorem by Ogawa, Uemura (2014)): Determination of a square integrable Wiener functional from its SFC-Ss
    - (S-D-2) Theorem (extension of Theorem (S-D-1) and another theorem by Ogawa, Uemura (2014)): Determination of a stochastic differential whose coefficients are square integrable Wiener functionals, from its SFC-Ss
    - (S-D-3) Theorem: Derivation of $a(t)$ from SFC-Ss of a stochastic differential whose diffusion coefficient $a(t)$ is a locally absolutely continuous Wiener functional and drift term $b(t)$ is square integrable almost surely
- (O) Identification from SFC-Os

(O-I) B-independent identification

(O-I-1) Theorem (extension of a theorem by Ogawa, Uemura (2018)): Derivation of the absolute value $|a|$ from SFC-Os of any noncausal finite variation process $a(t)$

(O-I-2) Theorem (extension of Theorem (O-I-1)): Derivation of $|a|$ from SFC-Os of a stochastic differential whose diffusion coefficient $a(t)$ is any noncausal finite variation process and drift term $b(t)$ is square integrable almost surely

(O-I-3) Theorem (extension of Theorem (O-I-2)): Derivations of $\mathrm{Re}\, a$, $\mathrm{Im}\, a$, $\mathrm{Re}\, a\, \mathrm{Im}\, a$ and $(\mathrm{sgn}\, a)a$ from SFC-Os of a stochastic differential whose diffusion coefficient $a(t)$ is the sum of any complex noncausal finite variation process and a complex local martingale, i.e. a process whose real and imaginary parts are local martingales, and a Skorokhod integral process and Hilbert-Schmidt integral transforms of Wiener functionals, and whose drift term $b(t)$ is square integrable almost surely

(O-I-4) Theorem (extension of Theorem (O-I-3)): Derivations of $\mathrm{Re}\, a$, $\mathrm{Im}\, a$, $\mathrm{Re}\, a\, \mathrm{Im}\, a$ and $(\mathrm{sgn}\, a)a$ from SFC-Os of a stochastic differential whose diffusion coefficient $a(t)$ is in a certain class $\mathscr{L}^{*,e}$ and drift term $b(t)$ is square integrable almost surely.

(O-D) B-dependent identification

(O-D-1) Theorem: Determination of a stochastic differential whose coefficients are square integrable Wiener functionals and whose diffusion coefficient is a Skorokhod integral process, from its SFC-Os

(O-D-2) Theorem: Derivation of any noncausal finite variation process $a(t)$ from its SFC-Os

(O-D-3) Theorem (extension of Theorem (O-D-2)): Derivation of $a(t)$ from SFC-Os of a stochastic differential whose diffusion coefficient $a(t)$ is any noncausal finite variation process and drift term $b(t)$ is square integrable almost surely

(O-D-4) Theorem (extension of Theorems (O-D-1) and (O-D-3)): Derivation of $a(t)$ from SFC-Os of a stochastic differential whose diffusion coefficient $a(t)$ is the sum of any complex noncausal finite variation process and a complex local martingale and a Skorokhod integral process and Hilbert-Schmidt integral transforms of Wiener functionals, and whose drift term $b(t)$ is square integrable almost surely

(O-D-5) Theorem (extension of Theorem (O-D-4)): Derivation of $a(t)$ from SFC-Os of a stochastic differential whose diffusion coefficient $a(t)$ is in $\mathscr{L}^{*,e}$ and drift term $b(t)$ is square integrable almost surely.

Here, in each statement above, the CONS $(e_n)_{n\in\mathbb{N}}$ which defines SFCs is taken generally and the above-mentioned result on the stochastic differentiability is used to obtain the results (O-I-3), (O-I-4), (O-D-4) and (O-D-5). Besides, we can get the same result on derivation from the SFC-Ss of a stochastic differential whose diffusion coefficient $a(t)$ is the sum of a function in $L^0(\Omega; L^\infty[0,1])$ adapted to the Brownian filtration and a Wiener functional $a \in \mathcal{L}_1^{1,2}$ such that $\sup_{t\in[0,1]} |a(t)|_{0,1,2} < \infty$, as (O-I-3), (O-I-4), (O-D-4) and (O-D-5) from the argument in their proofs. Moreover, all the results listed above on B-independent or B-dependent identification assert at the same time constructive identification in a certain assigned first-order language $\mathcal{L}_0$ or $\mathcal{L}_0^B$, respectively.

Journal:

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学位論文審査結果の要旨

学位論文題目

Stochastic differentiability, Ogawa integrability and identification from SFCs of random functions

（乱関数の確率微分可能性, Ogawa 積分可能性, 及び SFC による同定）

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本学大学院理学系研究科情報数理科学専攻博士後期課程４年に在籍の星野浄生氏は非因果的な乱関数についての確率解析を中心に研究を行っている。伝統的な確率解析の分野ではブラウン運動 $B(t), 0 \leq t \leq 1$ が与えられて乱関数 $f(t), 0 \leq t \leq 1$ について因果的（causal）という条件が課される。ここで因果的とは $f(t)$ の値がブラウン運動の時刻 t までの値 $B(s), 0 \leq s \leq t$ で決定されるということを意味する。$f(t)$ が因果的である場合に確率積分 $\int_0^1 f(t)\,dB(t)$ としては Ito 積分と Stratonovichi 積分という２種類のものがある。一方 $f(t)$ が非因果的である場合には Skorokhod 積分と Ogawa 積分という２種類の確率積分があり、それぞれ Ito 積分と Stratonovichi 積分の拡張であるとみなされる。星野氏の研究は主に以下の２つの問題に関するものである：

(A) Ogawa 積分の可積分性

(B) 確率 Fourier 係数による乱関数の同定

ここで 乱関数 $a(t), b(t), 0 \leq t \leq 1$ に対して $X(t) = \int_0^t a(s)\,dB(s) + \int_0^t b(s)\,ds$ と定め, $(e_n(t))_{n\in\mathbb{N}}$ を $L^2(0,1)$ の正規直交基底としたとき

$$(e_n, dX) := \int_0^1 e_n(t)a(t)\,dB(t) + \int_0^1 e_n(t)b(t)\,dt$$

を乱関数 $X(t)$ の確率 Fourier 係数（stochastic Fourier coefficient, SFC）と呼ぶ。SFC を定める確率積分が Skorokhod 積分か Ogawa 積分かに応じてそれぞれ SFC-S,SFC-O と表記する。SFC $((e_n, dX))_{n\in\mathbb{N}}$ が与えられたとして, 乱関数 $a(t), b(t)$ が SFC より復元されるかを問題 (B) では考察する。

星野氏は乱関数が Skorokhod 積分過程で表される Wiener 汎関数である場合に修士論文で得られた問題 (A) に関する結果（定理 5.1, 定理 5.2, 定理 5.3）をさらに広い Wiener 汎関数である場合に対して問題 (A) に関する結果を得た（定理 5.5）。また、乱関数が Wiener 汎関数である場合に修士論文で得られた問題 (B) の SFC-S に関する結果（定理 6.3, 定理 6.4）と SFC-O に関する結果（定理 6.7）を乱関数が局所絶対連続な Wiener 汎関数である場合の SFC-S に関する結果（定理 6.5, 定理 6.6）と非因果的有界変動過程である場合の SFC-O に関する結果（定理 6.9, 定理 6.10, 定理 6.11, 定理 6.12）を得た。これらの結果は小川重義氏と植村英明氏によって得られた問題 (B) に関する結果を適用される乱関数のクラスを有界変動過程の場合へと広げて一般化したもので単著の論文として専門雑誌に掲載が決定している。さらに正則な基底を用いて定義される SFC-O についての問題 (B) に関する結果（定理 6.13）を証明する際に用いられた $X(t)$ から $a(t)$ を導出する確率微分のアイデア（定理 6.4）は Ogawa 積分の提唱者である小川氏からも高く評価されている。

以上のことを踏まえて、本委員会は本論文を学位論文として十分な内容を持つものと判断した。

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