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学位授与の日付	平成 26 年 3 月 31 日	
論　　文　　名	「Stabilization of a Steady State in Time-delay Oscillators Coupled by Delay Connections (結合遅延が伴う遅延発振器群における定常状態の安定化現象)」	
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論文要旨

Oscillations are well known as important phenomena in nature and artificial systems. They can be classified as forced oscillations and unforced oscillations (i.e., self-excited oscillations). Self-excited oscillations occur in nonlinear autonomous systems, in which external energy is continuously provided, and their nonlinearity maintains their oscillations. Nonlinear autonomous systems with self-excited oscillations are simply called oscillators throughout this thesis. Some oscillations, such as the periodic firings of pacemaker cells, are beneficial to the performance of their various systems. Other oscillations, such as the wind-driven vibrations of bridges, harm the stability of their system and degrade its performance. In applications, it is imperative to avoid or suppress the harmful oscillations. In order to do this, the parameters must be chosen in such a way that the oscillations do not occur. This may require major changes in a system. An alternative is to suppress the harmful oscillations with feedback control; this is more practical, since it does not require major changes. There is an important method for suppressing oscillations: a delayed feedback control method can be used to stabilize an unstable steady state that is embedded within a system of oscillators, and it does this without direct knowledge of the steady state position. Thus, this delayed feedback control method has been useful for experimental situations, and it has been extended to various situations.

In daily life, we can observe synchronization, for example, in the flash of fireflies, the firing of pacemaker cells, and the chirping of crickets. In the field of nonlinear science, researchers have found many interesting phenomena relating to coupled oscillators, but if they are harmful (e.g., the oscillations in DC micro-grids), they must be suppressed. Amplitude death, the stabilization of a steady state that is induced by a static connection, is a strong candidate for suppressing the oscillations in coupled oscillators. Unfortunately, a static connection cannot induce amplitude death when the coupled oscillators are similar. In practice, this is a crucial limitation of the use of amplitude death. However, Reddy *et al*. showed that amplitude death can be induced in this case if the connection has a transmission delay.

Time delays inevitably exist in many dynamical systems, such as biological systems, traffic systems, and supply chains, since signal propagation and processing signals both have finite speeds. This may induce self-excited oscillations in nonlinear autonomous systems, such as metal-cutting tools, contact rotating systems, and oil-well drill-string systems. Nonlinear autonomous systems with self-excited oscillations that are induced by delays are called time-delay oscillators. Some oscillations in time-delay oscillators are harmful to the performance of the system, and it is desirable to suppress them. Stability analyses and the control of time-delay oscillators have gained increasing attention from a theoretical viewpoint as well as for practical applications; however, these are not easy tasks, since, due to the time delays, the dimension of oscillators is infinite.

Let us recall that previous studies dealt only with the stabilization of oscillators that did not have a time delay. However, we have seen above that the stabilization of time-delay oscillators is an important subject. The main purpose of this thesis is to apply the results of previous studies to the stabilization of time-delay oscillators. The contents of each chapter of this thesis are as follows.

Chapter 2 investigates amplitude death in time-delay oscillators that are coupled by a static connection. It has been reported that static connections cannot induce amplitude death in a pair of coupled identical time-delay oscillators. However, we have shown that the static connection can induce death when the oscillators have different delay times. Its stability analysis was a difficult task, since the two different delays in the characteristic equation prevent a conventional stability analysis. We have shown that this difficult task can be successfully performed by using the method known as the cluster treatment of characteristic roots, which determines the boundary curve of the region of amplitude deaths in the parameter space. However, for three coupled time-delay oscillators, the three different delays in the characteristic equation prevent the use of this method. We have shown that combining this method with advanced clustering and frequency sweeping allows us to obtain the boundary curves.

Chapter 3 considers a network of time-delay oscillators coupled by a delay connection. It has been shown that the stability of a steady state with uncertain topology is equivalent to that of a linear delayed system with an uncertain parameter. A simple sufficient condition for the steady state to be stable has been derived on the basis of robust control theory. This condition provided us with a systematic procedure for designing the connection parameters. This procedure has two advantages: the designed parameters can be used for any network topology, and the design procedure is valid even for oscillators with long delays. We used numerical examples to verify the analytical results for complete, ring, and small-world networks.

Chapter 4 shows that the multiple delay feedback control method can stabilize an unstable steady state in time-delay oscillators. We provided a simple systematic procedure for designing the feedback gain and the two delays in the feedback loop. The advantage of this method is that arbitrarily long delay times can be used for the stabilization. An electronic circuit experiment was performed to verify the stability region and the systematic design procedure. Furthermore, we have shown that a multiple delay connection can induce amplitude death in two identical coupled time-delay oscillators. A systematic procedure for designing the coupling strength and the two delays in the connection was provided. The advantage of the multiple delay connection is the same as that of the multiple delay feedback control method.

Chapter 5 considers steady-state stability in limit-cycle oscillators coupled by a digital delayed connection. The semi-discretization technique allowed us to derive a characteristic equation with real polynomials for which the coefficients depend on the network topology. Our numerical results proved that the digital delayed connection better induces amplitude death than does the conventional delay connection.

Chapter 6 summarizes our results.

審査結果の要旨

本論文は，結合信号ならびに発振器のダイナミクスに遅延時間を伴う結合発振器群の安定化現象を，非線形ダイナミクス・ロバスト安定理論の知見に基づき解析し，さらに，各パラメータの設計問題にも言及したものであり，以下の成果を得ている．

（１） 異なる遅延時間を伴う少数の発振器が，遅延を伴わない拡散的結合により安定化する現象の安定性解析を行った．さらに，これらの結果を，電子回路実験で実証した．

（２） 同一の遅延時間を伴う多数の発振器が，遅延を伴う拡散的結合により安定化する現象の安定性解析を行った．特に，ロバスト安定理論を駆使することで，発振器群の結合構造に依存しない安定条件が導出できた．この解析結果に基づき，発振器の遅延時間が長くても，安定化現象を誘発する結合パラメータの系統的な設計手順を示すことができた．

（３） 異なる遅延時間を伴う遅延フィードバックが，単体の遅延時間を伴うものよりも，遅延発振器の安定化現象に有効であることを解析的に示した．さらに，この解析結果を，2 個の遅延発振器群に適用し，安定化現象が発生することを確認した．また，これらの結果を電子回路実験によって実証した．

（４）遅延を含まない発振器群の安定化に,ディジタル遅延結合が有効であることを示した．安定性解析には疑似離散化法を活用し，この解析結果を数値シミュレーションで検証した．

以上の諸成果は，工学分野において，不安定化を誘発するために避けられてきた遅延が，工学システムの安定化へ積極に活用できることを示す基盤的知見を与えている．特に，大規模なシステムを念頭に置いて実施された研究であるため，得られた知見は，今後さらに複雑化する電力ネットワークや通信ネットワークの安定な運用に貢献できる可能性を秘めている．また，これらの成果は，申請者が自立的に研究活動を継続できる能力と学識を持ち合わせていることの証である．